The intension/intention of teaching mathematics

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In recent years mathematics educators have begun to draw on academic disciplines beyond mathematics and traditional, cognitive psychology as resources for theorising. Sociology, anthropology, and even psychoanalysis have formed the theoretical framework for some researchers. Mathematics education has become more conscious of culture, in all sorts of ways. But in cultural studies, writers have been addressing what it is to be and to communicate, and therefore to learn and to teach, in ways which of most of us in mathematics education have not been aware. In this paper I attempt to examine and use postmodern ideas to look, in particular, at mathematics teaching. (1)

Given the enormous changes in almost all aspects of life over the last two or three decades of the twentieth century, it is rather remarkable that the teaching of mathematics looks, for the most part, in most parts of the world, much the same as it did 50 years ago. From within the community it seems to suffer from an inertia that militates against substantial change in teaching styles or perceptions of learning. For instance, in the UK where we have national tests at ages 7, 11, 14 and 16, teachers say that they have no time for doing interesting mathematics, they have to just get on and teach the stuff. I understand and sympathise with classroom teachers for the pressure they are under but the point I wish to make is that it's as if the teaching/learning style doesn't make any difference: on the contrary the most efficient and time-saving is the familiar chalk-and-talk. Although the establishment of coursework as 30% of the marks in the national examinations in mathematics in the UK at age 16 in 1988 forced all teachers to integrate investigations and extended pieces of work into their mathematics teaching repertoire, for most this did not lead to a change in teaching style (Lerman, 1989). It rapidly became institutionalised into a set of standard procedures (Hewitt, 1992; Morgan, 1995) and in any case has now been reduced to 20% and become optional. From outside the community, governments, business, industry and parents hold onto an image of good mathematics teaching from a mythical past which is about rote learning, drill and practice, plenty of algebra, no calculators, and so on.

But if we look around us at what the world is like today we have to admit that we are in a time of rapid, almost breathtaking, change, what many call postmodern times: a new era of communications and access to information; of shifting employment patterns; potentially changing structures of government; and of global capitalism (Jameson, 1991). Culturally, socially and intellectually, the move to the postmodern has been rather like letting the genie out of the bottle. For the foreseeable future there seems no way of imagining how it might be possible to get it back into the bottle. Where one might have imagined that one could pin down, for example, objective educational aims, now one is forced to admit that there is no objectivity apart from a stance, a position, bringing with it a situation of powerfulness or powerlessness. Take the examples of culture and communication. They must be seen today as commodified and political. What constitutes culture is what commands high prices and is valued popularly, the constant search for the new whilst colonising the old, the debates about high and low culture; all these are about markets, success, fame and consequently power. They owe little to Gombrich's and Read's debates about Art with a capital A. Those who own the channels of global communication and control who has access to those media, also control what we know about the world. Baudrillard could claim that the Gulf War never 'happened', it was just a souped-up computer game. The anarchism of the World Wide Web may fragment late capitalism's increasing domination of communication and the

media, but it is by no means certain. There seems no way of turning back from the condition of postmodernity (Harvey, 1989).

What about education in the postmodern world? "The very possibility of education depends on arriving at some view about how people and societies can and should be *represented*." (Kemmis, 1995, p. 136) If we are forced to recognise that all such representations will be partial, that they will be readings, and can even be described as simulacra, fictions which will give access to some and deny it to others, then there is a need to review what we mean by education, curricula and for our particular concern, the teaching of mathematics. I have used words of coercion, such as 'forced' and 'admit' when speaking of postmodernity because it feels for me and for many others as a wrench from a time of at least the possibility of certainty, of being convinced that one can speak of, in particular, class as a fundamental structure of society, but also of culture as something hard to define but nevertheless reachable. It appears not to be as painful for our children who are growing up in this era, and it has some interesting repercussions. For instance, in the UK in higher education, having gone through the 1980s where students chose courses for the secure financial future which these courses seemed to open up, security which often turned out to be a mere chimera, they are now changing to philosophy, psychology and sociology. I interpret that as being about styles but also as about students recognising the inherent unpredictability of their futures and the impossibility of determining what the economy wants and needs even three years hence, and so it becomes instead a choice of doing what looks interesting now, and facing whatever comes next when it arises. I feel sure that the uncertainty is heightened by the *fin de siècle*, indeed the end of the millennium.

Twelve months ago Shlomo Vinner (1997) warned the PME community that we ignore developments in children's lives, in education and in mathematics at our peril. Focusing here particularly on teaching, there is no doubt that we face major problems: in the UK we continue to fail mostly working class children and some ethnic groups in terms of acquiring certification in their mathematics; each year there are less interested and/or qualified people coming into mathematics teaching; the profession has a low status which goes together with generally low pay when compared to other professions, mathematics remains probably the most unpopular school subject; and in adult life people will happily own up to innumeracy, whereas illiteracy is something about which to be embarrassed. It has been suggested that the UK government's move back to basics in numeracy and literacy is a doomed attempt to plug the postmodern bottle again, to lay claim to the established verities of the foundation of knowledge as supplied by the three Rs, but unfortunately the genie has already escaped. It is certainly, however, an effective way of controlling and disempowering teachers. In the last year, since the election of a Labour Government, the Department for Education and Employment (does the the name implies the aims for education?) has issued pronouncements on: calculator use (or rather non-use) in primary schools; the required time for homework; the compulsory implementation of a numeracy hour (equal to 50 minutes!) every day for primary pupils; and the correct style of teaching (interactive whole-class) to be used. With the implementation of a National Curriculum for mathematics in primary and secondary teacher education courses next academic year the intention of many of us to present the image of mathematics teachers, educators and researchers as professionals will be undermined even further.

I want to take this opportunity to cast a concerned eye, from within the community, over the mathematics classroom. First I will review what postmodernists are saying about mathematics and about teaching and I will then take a sceptical look at what we are doing in much of our research on teaching and teacher education. Finally I will suggest some moves that we might make in response. I do not claim to be able to offer any answers: indeed it would be inappropriate to a paper on postmodernism even to try. My aim is merely to try to ask some useful questions about what might be *meant* by the term 'teaching mathematics', which is the sense of the term 'intension' in the title, and how one might encourage and enable the *becoming*-teacher, the sense of the word 'intention'

in the title. I will play with these two words a little in this paper. A struggle with words and meanings brings a self-consciousness about language use to the surface, an important element in postmodernism. I am trying, also, to articulate and make explicit a struggle over what is taken to be 'teaching mathematics' (intension), which is usually implicit in mathematics education whilst it determines what people want to do or want done in teaching (intention).

Academic Mathematics

In the early years of my work in mathematics education I was very preoccupied with a characterisation of views of the nature of mathematics as either relativist or absolutist and the possible pedagogic consequences of the adoption of either of those two positions. Such a binary opposition is outdated in a time of postmodernism. I will try to sketch a view of the development of mathematics which owes more to Wittgenstein's notion of language games than to relativist/absolutist oppositions.

The invention of non-Euclidean geometries in the 19th century had a profound effect in mathematics and in philosophy. Euclid's fifth postulate appears to have worried him, and many other mathematicians, for centuries afterwards: its very length suggests that it is more problematic than the others. Euclid seems to have structured the Elements to start with as many theorems as possible, the first fifteen, which do not require that postulate (Kline, 1980). Centuries of attempts to resolve this concern followed, including rephrasing the postulate to present it as less open to doubt. One version of it, due to Playfair, is that through any point not on a given line there exists one and only one parallel. Others tried to prove it from the first four postulates. In attempting to prove the fifth postulate by *reductio ad absurdum* Bolyai, Lobachevsky and Gauss separately found that they could not reach an absurdity; the fifth postulate is an independent postulate. You can take it in Euclid's form, or another opposing version of it, such as that there is more than one parallel. Either way, the resulting geometry is quite consistent. Instead, therefore, of having a single geometry of nature, fitting empirical phenomena, as given by God the Great Geometer through the way the universe was constructed, we now have many geometries. It is not that one is right and the others wrong as far as the world is concerned: there is no way of saying which geometry one should use *a priori*. The geometry chosen structures the problem and the potential results. Einstein (1954) chose four-dimensional Riemannian geometry for relativity because it suited his hypotheses about matter and its distribution. "Hilbert spaces are symbolic abstractions, not geometric pictures. Although Hilbert spaces are rule-governed, movement within these spaces is more like a move in a game of chess than a knowledge-claim" (Ashley & Bettebenner, 1996, p. 140). In this period of what they term late modernity, using a framework given by Baudrillard (1983), mathematics had become a game which marked the absence of any discernible referent, a dissimulation. That is not to say that it is meaningless, merely foundationless. It is about rule-governed activities within which meanings are carried, meanings which certainly matter greatly to the people engaged in them. The dissimulation forces us to focus on the rule-governed discourse rather than on what and whether it represents. Beginning with non-Euclidean geometries and culminating in the twentieth century failures of the foundationalists' struggles, the Kantian image collapsed.

Moves into hypermathematics, starting perhaps with the reluctant acceptance into mathematics of Appel and Haken's 1977 proof of the four-colour theorem, can be said to have marked the postmodern. The problem, that any map can be coloured using four colours only, was reduced by them to just (!) 1,482 configurations, but the evidence of the validity of this reduction and the checking of those cases took 1,200 hours of computer time and 10,000 pages of paper copy (Tymoczko, 1986). Lakatos (1976) argued that mathematics is an *a posteriori* science, in the sense that examples can be counter-examples leading to revision at a local or global level; the trouble is that the evidence in the case of the four-colour problem is humanly unsurveyable. We could go further and claim that "If a whole branch of mathematics were to be based on

hypermathematical computation, this sub-discipline would to some extent be unsurveyable, auto-constitutional, and self-referential" (Ashley & Bettebenner, 1996, p. 148). In chaos theory we have seen how the determinism of a set of rules is profoundly affected by unpredictable initial conditions, yet patterns almost miraculously emerge. Thus, in weather prediction meteorologists now produce perhaps 50 different forecasts, arising from different starting conditions, different vigour in the flapping of the butterfly's wings, and the pattern that emerges produces a reliable range of possibilities. Finally, there are only the rules of the language game, but following the grammar still leads to fascinating and often useful results.

I do not want to suggest that the transfer of this view of mathematics into the classroom is easy, or necessary. Clearly a recontextualisation is effected through principles whose nature must be addressed (Bernstein, 1996). I want to emphasise, though, that an argument can be made for a mathematics that is a very different animal to the one of popular perception. It is not a question of whether the animal has two or four legs, but perhaps that it has no legs at all, or body, but is merely a simulacrum.

The Individual Child

Mathematics teaching still focuses on facilitating the individual child's cognitive constructions, and much theorising in mathematics education goes into creating a language for describing and creating a child-centred, constructivist practice (for a critique of constructivism see Lerman, 1996). This is the dominant perspective even though the image of the individual as the source of sense-making and as the autonomous builder of her/his own subjectivity has been hardly a tenable claim since it was challenged by Marx and Durkheim in the nineteenth century. The extent to which the individual is constituted in and through economic, social and cultural forces has been the subject of much discussion and analysis outside of the mathematics education community, and some within (Evans, 1993; Evans & Tsatsaroni, 1994; Klein, 1997a; 1997b; Lerman, 1998a; Winbourne & Watson, 1998). Today we might want to talk of the individual as a fragmented self at the intersection of a unique collection of overlapping identities constituted in different practices, as lived out through class, race, ethnic, sexual, gendered, regional and other positions. Thus "we cannot fully specify the psychological subject/agent as an object whose nature can be defined in isolation from a context" (Harré & Gillett, 1994, p.26). This often seems almost contradictory, since the language available to us to talk about our minds is one which gives the appearance of a context-independent inner reality. However Wittgenstein's discussion of pain puts another perspective on language and reality, in particular the nature of our internal, mental reality. Bloor (1983) describes it thus:

> "One possibility, he *(Wittgenstein)* said, to explain how we learn the word 'pain', is that 'words are connected with the primitive, the natural, expressions of the sensation and used in their place' *(PI, I, 244)*. A child who hurts himself and cries is comforted by adults who provide a vocabulary for expressing the pain. The word 'pain' does not mean 'crying' but takes over from it. The newly acquired verbal behaviour, he says, is itself a form of pain behaviour." (p. 51)

Labov's experiments from 1972, replicated and elaborated by Cole and colleagues in 1978, offer some powerful examples of children speaking very differently about a situation or task, depending on the social setting, the other persons in the interaction, and the perceived goals, demonstrating the fragmented nature of subjectivity. Labov describes an African-American child, thought to be deficient in language, demonstrating that 'deficiency' when talking with a white interviewer in a formal setting and with a black interviewer in the child's home. In a third situation the black interviewer sits with the child and his friend, eating snacks, and the interviewer introduces taboo words and topics. "The effect of these changes on Leon's speech was dramatic. Not only did he go beyond one-word replies to questions, but he actively competed for the floor... Labov

concludes that Leon had no difficulty using the English language." (Cole, 1996, p. 236). Of course Nunes, Schliemann and Carraher's (1993) work in relation to the significance of the socio-cultural setting and the nature of the activity on mathematical performance, and the consequent misperception of children's abilites when judged soleley by mathematics in the school setting is well known, as is that of a number of others (Saxe, etc.).

The situation-specificity of children as persons, then, raises major questions for teaching and learning mathematics. What is it to learn? Can one speak at all of decontextualised knowledge of mathematics? How can one use the results of cognitive studies? Is it perhaps more useful to speak of aiming at children becoming school mathematics persons, in much the same way as children become part of other social and cultural groups, with their own language, rules and logics? I will return to this idea below.

Teaching

One of the most significant moves in thinking about teaching (one might say one's intention in teaching) in the last few decades has been that of reflective practice and its association with critical theory and action research. Reflective practice offers a view of how teachers act in the classroom as informed, concerned professionals and of how they continue to learn about teaching and about learning, about themselves as teachers and about their pupils as learners. It encourages and feeds the notion of the autonomous, emancipated teacher who is not dominated by government rhetoric in choosing the way s/he wishes to conduct the classroom interactions, nor by the self-interests of some university-based researchers in defining what constitutes valid research. At the same time its association with critical theory invites an engagement with the institutions of schooling in actions to change those structures and emancipate thinking and acting. It injects a relativism into what we can know about teaching: in this classroom, with these students, this learning material, certain things happened which might be explained thus and might be acted upon thus. It seeks to avoid the traps of extreme relativism, however, by emphasising principled thinking, reason, and critical judgement. The critical, reflective practitioner:

> "thinks and acts in accordance with, and values, consistency, fairness, and impartiality of judgement and action. Principled, critical judgement, in its rejection of arbitrariness, inconsistency, and partiality, thus presupposes a recognition of the binding forces of standards, taken to be universal and objective, in accordance with which judgements are made." (Siegel, 1988, in Parker, 1997, p. 44)

It is precisely here that the critique of the postmodern enters. First, there has to be a pre-supposition of autonomy in order to argue for autonomy. Unless one is free and emancipated, how can one recognise when the dominated teacher steps out from their chains, whether that teacher be oneself or someone else? From which Archimedian position can one identify that autonomy? Positivists appeal to the transcendental to resolve this, that the truth of the transcendental argument presupposes even being able to ask the question. Habermas uses the ideal speech situation in this way: "all speech, even intentional deception, is oriented towards the idea of truth... Insofar as we master the means for the construction of an ideal speech situation, we can conceive the ideas of truth, freedom and justice" (Habermas, 1970, in Parker, 1997, p. 58). Similarly, rationality is claimed to be a transcendental notion. To argue about rationality is to presuppose rationality. The problem is that reasoning is a process, a language game, whilst what counts as justification for reasons, or validity of reasoning, is specific to a social context or is overlapping across a range of social situations and is entirely part of the grammar of the particular language game. The force of a parent's statement as reason to a child, "Because I say so", is in the nature of the relationship. The child, and perhaps later in the day the parent too, may well recognise that it was not a reasonable reason,

and certainly not one that had been justified. Such a statement would certainly not be considered reasoning in a debate. Rationality, then, is deconstructed, it needs to be placed under a sign of erasure, what Derrida calls *sous rature*, and written ~~rationality~~, which highlights the absence of a foundation whilst it claims to provide a foundation; so too with ~~autonomy~~.

Second, from where is the criticism, rejecting arbitrariness etc., to come? Views are positions, perspectives from where one is situated. A critical view is another view. Now the claim of critical theory is that one can become aware of how one's intentions may have become distorted by self-interest or by false ideology, and one can reject and remove these distortions:

> "a critical social science will seek to offer individuals an awareness of how their aims and purposes may have become distorted or repressed and to specify how these can become eradicated so that the rational pursuit of their real goals can be undertaken" (Carr & Kemmis, 1986, p. 136)

There is no doubting that each of us has aims, purposes and goals but, first, they are always context-specific and, second, what might it mean to describe them as distorted or repressed? Indeed the goals of critical theory have been seen by some as far from emancipatory. Reporting on a course designed to address experiences of students' sense of powerlessness on a US campus, which began with a critical theory approach but then adopted a post-structuralist one, Ellsworth (1989) made the following points:

(i) It appears that critical pedagogy draws on a notion of a decontextualised individual when encouraging teachers to help students to identify and choose between "sufficiently articulated and reasonably distinct moral positions" (Liston and Zeichner, 1987, in Ellsworth, *op cit.*, p. 304). The argument for choosing rationally between moral positions is again based on treating 'rationality' as a transcendental signifier. Further, it can be understood as operating to establish an irrational Other and therefore denying a voice to perspectives that question the positions guaranteed by rationalism (p. 306). In contrast, poststructuralist thought is not bound to reason, but "to discourse, literally narratives about the world that are admittedly *partial*. Indeed, one of the crucial features of discourse is the intimate tie between knowledge and interest, the latter being understood as a 'standpoint' from which to grasp 'reality'". (Arnowitz (1987/88), in Ellsworth, *op cit.*, p. 304)

(ii) Some critical pedagogues argue for the teacher seeing herself as re-learning with the students, thus equalising the teacher/learner imbalance. Nevertheless, Ellsworth suggests that teachers are expected to use this equality to liberate and empower their students, thus re-emphasising the imbalance in knowledge and power (p. 307).

(iii) The authority of the teacher is justified as 'emancipatory authority' in that it is to be continually questioned and critiqued by the students and the teacher, and it is maintained through the respect and trust of the students. Ellsworth points out that the multiple socio-cultural positionings of the actors in the classroom setting are characterised by shifting relationships of power and powerlessness, of voice and its lack, through the many overlapping and separate identities of gender, ethnicity, class, size, age, etc., to say nothing of the 'unknowable' elements of the unconscious (p. 307-8, 318). Unless these positionings are given voice, 'emancipatory authority' is as repressive as any other.

(iv) No matter how strong one's identification with other oppressed groups, one can never know what it is like to be an other.

> "My understanding and experience of racism will always be constrained by my white skin and middle-class privilege. Indeed, it is impossible for anyone to be free from these oppressive formations at this historical moment. Furthermore, while I had the institutional power and authority in the classroom to enforce "reflective examination" of the plurality of moral and political positions before us in a way that supposedly gave my own assessments equal weight with those of students, in fact my

institutional role as professor would always weight my statements differently from those of the students." (p. 308)

Feminist research, as well as postmodernism, has encouraged us, researchers and writers, to recognise and speak out our positionings and our powerfulness and powerlessness. In this way we at least share the difference and the fragmented nature of our overlapping subjectivities (Lerman, forthcoming, a).

There is no time or space here to pursue further the lengthy debates about postmodernism and critical theory (see e.g. Kemmis (1995)), and in particular the responses to Ellsworth's paper (e.g. 'Correspondence', 1990). Ellsworth and others express powerful concerns about critical theory. I will argue below, however, that one can continue usefully to draw on notions of reflective practice but from a different position.

Klein (1997a) describes similarly a shift in her teaching in which she saw her "practice as being coercive in that students always had to arrive at the authoritative 'truth' as portrayed throughout my subject and discriminatory in that students not adhering to my construction of the 'autonomous' student were classified as unmotivated" (p. 291). In this case Klein's strongly held theory was not critical theory but constructivism, whose tenets she was attempting to encourage student teachers to learn and adopt. She found that a poststructuralist focus enabled her to critique her practice and offered her a language for reconceptualising the function of the support that teachers give to move away from coercion to a position of enabling students' voices.

Research on Teaching Mathematics

In previous decades, as may be seen for instance in the early years of PME proceedings, it was thought that the answers to research questions concerning the teaching of mathematics could be found within mathematics itself (either diachronically, as in the search for epistemological obstacles, or synchronically, as in establishing structural/logical connections between concepts) or in developmental and cognitive psychology. In the last ten years, however, research on teaching and learning mathematics has diversified considerably. I will review here some of these directions. I believe we will see the same kind of fragmentation and lack of foundations which characterises the previous sections.

Teachers' Beliefs

I will start with research on teachers' beliefs, that is their theories about teaching (one might say the intension of teaching for them), their theories about practice, and their teaching as observed by researchers, work in which I have played a part (1983, 1986, 1990). It has been argued (see, e.g. Thompson, 1984) that teachers' beliefs are critical factors determining how they teach. So-called mismatches between theories and practices have been discussed in the literature (Cooney 1985; Thompson 1982, 1984, 1992; Lerman 1986, 1990), although it is sometimes referred to as espoused and enacted theories of mathematics teaching (Ernest 1989). In her review of the research in this field, Thompson (1992, p. 138) suggests that the relationship between teachers' conceptions of mathematics and their practice is complex and argues for viewing the relationship as a dialectic one, citing the work of Cobb, Wood & Yackel (1990). Mason (1990) engages with this dialectic by arguing that through awareness the teacher can reconcile "the theorist and practitioner inside" (p. 178). Whilst I feel sure that the relationship is a dialectic one for each of us in our teaching, in terms of researching theories and practices I want to suggest that the problem lies elsewhere. Research which examines teachers beliefs and theories in one context and attempts to examine practice, or beliefs about practice, in another context is again based on a notion that the core of a subject's identity is somehow unified and decontextualised. It is as if the teacher brings theories (abstract mental objects) to bear on practice (an empirical domain of actions affected by emotions), although it may be described as theories influencing one's practice and practices influencing one's theories (Thompson's 'dialectic'). Such a description

recalls the classical philosophical distinction between 'knowing how' and 'knowing that'. Given this separation, attitudes and beliefs about the teaching of mathematics can be examined by an instrument in one setting, interviews in a laboratory or questionnaire completion on one's own for example, and their impact examined in another setting, the classroom. The activity is seen as, in essence, the same. There is, then, the argument that whatever mismatches there appear to be result from the influence of one particular factor, in a school environment perhaps, that distorts or over-rides beliefs.

These researchers recognise that the shift from one setting to another allows the appearance of factors that significantly change teachers' actions from those they would profess to apply or would wish to apply but there is no clear sense of any mechanism or relationship between settings and actions and/or beliefs. I would argue that, whilst there is a 'family resemblance' between concepts, beliefs and actions in one context and another they are qualitatively different by virtue of those contexts. One cannot speak of decontextualised opinions or actions; the setting in which the questions are asked constitutes the conversation and is not separable from it. The activities of answering a questionnaire, talking with a researcher, or teaching, are not, in essence, the same. Essentialism is not viable as an underlying supposition of human social behaviour.

Mathematics Teacher 'Change'
Research on teaching has fed into teacher education courses, both in-service and pre-service and back into research on teacher education. Interestingly, it is usually termed research on teacher 'change' rather than education or learning (Lerman, 1997). There have been some very impressive projects aiming at teacher change, described and analysed in Clarke (1997), including the well-known Cognitively Guided Instruction project and the Purdue project in the USA. In the many reports I have read it is clear, from the interviews with teachers, that they consider that their practice has changed, sometimes dramatically. I include here also the smaller scale studies such as Davis (1997) and Clarke's own (1997) project. My comments are not to criticise this work but to suggest a different reading.

The notion of 'change' suggests a teleology, a direction from something to something else, a desirable goal. As mentioned above, it is not generally called 'learning', except perhaps in pre-service teacher education courses. It may be that, in projects such as these and on in-service courses, it might sound too prescriptive or even arrogant to claim that teachers may be learning something about teaching. Nevertheless projects, particularly those motivated by the current reform in the USA, do have a vision of better teaching, teaching that, it is believed, will enable more students to learn more mathematics. Is it surprising that teachers change their perceptions of their teaching and in many cases are seen by the researchers to have changed their practice? As alternative stories of what teaching is (intension) and how it might be are offered to teachers, who have taken on board the goals of the project at least at the level of a commitment to participate, it might be expected that they would appropriate some of these stories and 'become' different teachers. I have in mind an analogy with a different setting, that of psychoanalysis where, in some interpretations, the patient is helped to develop different stories for their behaviour and experiences, some of which become meaningful, that is to say the patient becomes the person in those stories. A particularly vivid illustration of this, for me, comes in Davis's (*op cit.*) study where the researcher and the teacher develop a way of characterising teaching, through the two years of the teacher's development or change, as different forms of listening, termed evaluative ("the complement of listening strategies" (p. 360)), interpretative ("being aligned with constructivist theories of learning" (p. 364)) and hermeneutic ("to reflect the negotiated and participatory nature of this manner of interacting with learners" (p. 369)). My reaction was to realise that the research offered me, as it did them, a new story for thinking about teaching as that teacher's intentions for her teaching changed. It is not that teaching is listening, or that what is being suggested, namely that it is helpful to focus on how one listens, prescribes good teaching.

Clarke's (*op cit.*) study reveals factors that influence change, as perceived by the participants in the project, emphasising in particular innovative curriculum materials and the relationship between the researcher as critical friend and the teacher. Another direction of research on teacher change is the growing interest in using Shulman's (1986) description of a teacher's knowledge being about both subject matter knowledge (SMK) and pedagogic content knowledge (PCK) (Even 1990; Ponte, 1994). They are not independent of each other and the most interesting research emerges from considering their interaction. Even, Tirosh and Markovits (1996) describe how they use this interaction between PCK and SMK in teacher education programmes. For example (p. 126), they offer pre-service teachers possible student answers to the addition of fractions and the teachers engage with issues of teaching at the same time as sorting out for themselves the why of fraction addition.

I began this section by referring to research projects whose goals are to reform teaching in a certain direction and I then moved to those from which I gain most as reader, those that study teaching development whilst theorising about teacher development, without an explicit notion of a better way to teach, offering new readings of teachers and teaching mathematics. It seems to me that many of these researchers are not seeking closure, the assumption that ultimately there are answers to what is good teaching of mathematics and that reasoning and/or research will reveal the truths. Postmodernism is characterised by the denial of the possibility of closure, of an ultimate correct account, and instead encourages the production of a multiplicity of accounts with their meaning and value established locally.

Recontextualisation
School mathematics is a different animal from academic mathematics. It is subject to a social process which Bernstein calls recontextualisation (1996, p. 24). It is also different from everyday mathematics which is often harnessed, as a resource, into school mathematics. We can identify and discuss different expressions of school knowledge, or we can call them different levels of school knowledge, such as the construction of the school curriculum, the production of textbooks for schools, or that of school practice, in our case teaching mathematics. There is also a need to distinguish between mathematics education as a field of inquiry (Zevenbergen, 1996; Dengate & Lerman, 1995; Ernest, 1998) and the practice of teaching school mathematics. They are strongly linked, of course, and it is certainly the case that when we are engaged in the field of mathematics education we hope that we will have some effect on practice. Bernstein calls them the field of production and the field of reproduction, with the field of recontextualisation between them (1996, p. 116). The point is to recognise that the principles which govern the process of recontextualisation, at any time and any place, are arbitrary and serve interests and purposes. Such a realisation enables a re-inscription of those principles, not in the expectation or demand that those principles will be emancipatory, free of bias and undistorted, but principles which arise from local commitments and beliefs shared by the participants in a community of practice.

A Postmodern Pedagogy - The Intension of Intended Teaching
It is not that postmodernism means a rejection of reflective practice or the possibility of change in one's classroom, school, university or region. It means a re-inscription of these activities into a language of the recognition of difference; of a rejection of the expectation of closure but not of a loss of meaning and values; of deconstruction of

reason, rationality and emancipation. Ellsworth (*op cit.*) argues that a classroom based on a practice

> "grounded in the unknowable is profoundly contextual (historical) and interdependent (social). . . What remains for me is the challenge of constructing classroom practices that engage with the discursive and material spaces that (the removal of the privileged self-image of the critical pedagogue) opens up" (p. 323).

The grand meta-narratives have not served us well. We are subject to capitalism's manipulations more than ever: racism, sexism and other forms of prejudice are increasing; nationalisms which oppress and murder others proliferate. Education, certainly in the UK and I am sure in many other places, is ever more dominated by the language of the market: measurement, standards, competencies, performance criteria, raising standards by tests of children, teachers and schools and by threats to teachers and schools. Rather than aiming for equality and emancipation we may be better served by recognising that the multiplicity of the social and cultural identities which constitute each of us result in powerlessness and powerfulness as those fragmented identities surface. We can create and celebrate pedagogies, theories and styles which allow and encourage the expression of differences and lead to the recognition that change at the local level is both possible and, actually, all there is. For example, in an earlier paper (Lerman, 1994) I referred to the most commonly used set of British textbooks, the SMP (School Mathematics Project), in which there are 4 levels of books for levels of ability (see Dowling, 1998 for a detailed analysis). I suggested there that one could offer pupils examples of the 'same' topic from top and bottom ability books to engage them in a critique of the way that notions of ability are constructed, and of how they themselves are constructed as able or not by the texts (Dowling, *op cit.*). This is not, however, a claim that there can be such things as textbooks free of bias, but that local action and deconstruction of the texts that construct people's identities and delimit their futures are the only source of taking power.

The point is that doubt and criticism, the cornerstones of critical reflective practice, don't 'bottom-out' upon arrival at freedom from bias (Parker, 1997), but at the language games in which one validates 'true', 'acceptable' and 'good'.

> "I did not get my picture of the world by satisfying myself of its correctness; nor do I have it because I am satisfied of its correctness. No: it is the inherited background against which I distinguish between true and false." (Wittgenstein, 1969, section 94)

Thus rehabilitated reflective practice is local, an expression of belief and commitment of the teacher(s) for improvement in teaching and learning. It is not that the binary oppositions of rational/irrational, emancipated/biased, illusory/true are wrong, merely unaskable and not interesting (Parker, *op cit.*, p. 141) in postmodern pedagogy. The issues to be tackled are meaningful at the local level.

Teaching and Learning
There is a sense of the intension of teaching in which it is a precondition of learning. It seems to me that there is a common thread, from Durkheim and Marx, through Vygotsky, to Wittgenstein and postmodernism, whereby all of what one might understand as human consciousness is the product of communication, social interaction and the relations of historically developed, socio-cultural experience.

> "Grammar tells us what kind of object anything is." (Wittgenstein, 1958, 373)
> "*Essence* is expressed by grammar." (*ibid*, 371)
> "How do I know that this colour is red? - It would be an answer to say: 'I have learnt English'." (*ibid*, 381)

People will not learn things on their own, by which I mean that a child born without human contact, the mythical 'wolf child', will not become a conscious human being. To develop conscious attention, memory, awareness and to gain culture, knowledge etc. are all the results of learning from others. "Instruction and development do not meet for the first time at school age; rather, they are in fact connected with each other from the very first day of a child's life." (Vygotsky, 1956, in Wertsch, 1985, p. 71). Harré (1997, personal communication) gives an example from Bruner's work of how this process operates from the earliest days of a child's life:

> "Bruner was interested in the very early moment of the development of intentionality and causality. Here's an attractive object. And an infant, a very small infant, reaches for it. Causal relation. Object is there...infant reaches out like that. What happens? Mother gives it to him. At that point the relationship between the reaching and the grasping changes...the mother has introduced into this game, baby wants, baby intends. Baby just acted causally...didn't get, so mother says baby wants. Next step, so she's completing the action, she is giving him the interpretation of his action. Next step, baby reaches forward, and compresses the air in his little chest and goes, 'Ah.' So, mother gives it to him. We've now turned the intentionality into a babble, the first primitive verbal utterance. Causality turning into intentionality. And the mediating role is the caretaker, who has done the defining, and done the supplementing." (2)

The learning may be in everyday situations, or in school or other intentional learning situations. The distinctions between them are important but the similarities are just as important. In both cases there is a necessary imbalance, one that is inherently about power relations, whether it be the social group whose rules the child wants to acquire in order to belong, or the classroom in which there is the struggle between multiple social demands on the child, of which a major demand is that of the teacher. Of course at a later stage one can learn from books, which one apparently does alone, but books are the writing down of human life and the learner-from-books has also learnt how to learn from books.

Now I do not want to suggest that teaching necessarily leads to learning. In some recent analyses of classroom videos of young children in a nursery classroom (Meira & Lerman, forthcoming) we have been describing situations where teaching, in the sense of the teacher's intended actions in the classroom, does not always lead to learning. We have described this as the zone of proximal development (zpd) not being created. We interpret the zpd as a creation of the interaction, including the goals and needs of the actors, the social relations of the classroom, the texts, the framing of the task and so on. The zpd is the classroom's (3), if it is anyone's (it is often assumed to be a kind of force-field which the child carries around, the dimensions of which the teacher must gauge); hence the inherent unknowability to which Ellsworth refers. The teacher and child may well miss each other's meanings. Similarly in other work I have been carrying out analysing peer interactions (Lerman, forthcoming, b) students may not catch each other's ideas (Vile & Lerman, 1996) and hence not create a zpd. Creating a zpd is as much about mutual alignment of goals and desires as about the intended content of the interaction and paying attention to this allows the potential for the re-inscription of teaching and learning into a language of difference which I emphasised above.

Learning as Becoming
Studies of how people learn in out-of-school situations, particularly work environments (Lave & Wenger, 1991), have offered valuable perspectives of the nature of the alignment of goals and desires by the learner and the teacher, the latter as mentor or 'master' (I use this gendered term in the absence of a more suitable one). In order to bring those notions into her discussion of schooling Lave (1996) draws on Olsen's study of the way that

schooling shapes the identities of newcomers to the USA in terms of the "racialization of social relations and identities" (p. 159). Describing learning in terms of students becoming, in our case, motivated participants in school mathematics, it seems to me, is where Lave's approach is particularly fruitful for us. Lave's focus on the shaping of identity in social practice emphasises the centrality of the social relationships constituted and negotiated during classroom learning. Lave talks of learning as "an aspect of participation in socially situated practices" (p. 150). Provided we do not expect those practices to be those of the teacher, in our case of mathematics, or the practices of the mathematician, but instead of the practices of the particular mathematics classroom culture, seen as the intersection of a multiplicity of practices, this interpretation of learning is very useful, as is shown by Winbourne (1997), and Winbourne & Watson (1998) for example. After all, for the most part, students do not want to become teachers of mathematics, nor mathematicians, and hence the direct transfer of the apprenticeship model into the classroom by some writers is perhaps too simplistic.

Learners come to the classroom as persons of multiple, overlapping subjectivities. Different aspects of those subjectivites are called up by different aspects of the practices of the classroom, and are expressed through identities of powerfulness or powerlessness. At the same time, new subjectivities are constituted in the social relationships and forms of communication which make up the activities of the classroom. Rather than the intension of teaching mathematics as the handing over, or the individual construction, of ultimately decontextualised mathematical concepts by the teacher or by the pupil respectively, teaching can be conceived of as enabling pupils to become mathematical actors in the classroom and beyond. The goals and needs of pupils, and the ways of behaving and speaking as mathematicians, become the focuses of the teacher's intentions. This does not translate into either side of a traditional-drill-and-practice/progressive-group-learning dichotomy, however. Both may well be appropriate at different times. That dichotomy, like so many others, are irrelevant and uninteresting; clearly so when seen from a postmodern perspective.

Social Semiotics of Mathematics
Objects, including concepts, have meanings only within relations of signification (Walkerdine, 1988). One of the familiar examples of this is workers not seeing their work practices as mathematical although a mathematician looking at those practices would wish to say that they can be seen as applications of mathematics (e.g. scaffolders using lengths of pipes which are Pythagorean triples). The issue is to examine and identify from within which practice one is observing, since those practices carry with them their relations of signification. An illustration of this might be the following: Take a class of young children for a walk around the neighbourhood with the instruction "Observe". Repeat the walk, this time with the instruction "Observe and identify as many examples as you can of circles, triangles, squares and rectangles". The second activity is different from the first because the children are becoming different actors, they are observing the 'same' objects with a different pair of spectacles, those of the mathematician. From the perspective of teaching and learning mathematics the research programme would therefore be to study empirically the semiotic mediation of those objects. The language of semiotic mediation, whereby the person and the world for that person are transformed by the acquisition or appropriation of cultural tools (analogous with Marx's thesis concerning physical tools) which are historically, socially and culturally constituted artefacts, is a resource which engages with the specificity of relations of signification (Lerman, 1998b). It offers a medium through which one can account for: cultural specificity, such as the understandings of the Aboriginal student described in Klein (1997b), both of mathematics and of appropriate teaching styles; the effects of particular IT tools, such as using a dynamic software package for learning geometry; the manner in which learning the place value system structures perceptions of number; and the importance of enabling the articulation of students' own understandings of ideas and concepts.

The Intention of Teaching Mathematics

I will conclude this paper with some remarks about mathematics teacher education, drawing again on the second spelling, intention, in the title. Many teacher educators will be familiar with the problem that, despite sometimes writing good theoretical essays on teaching and learning during a pre-service course, students often begin teaching in the same way that they would have without attending the course (Crawford & Deer, 1993; Lerman, 1997; Klein, 1997b). It appears that courses do not provoke students to confront their naive notions of teaching mathematics. It may better be expressed in Lave's terms, as I discussed above, that student teachers have a sense of who and how they will be as teachers before coming to the course. The ideas that we offer, and even the essays that the students write to which we give credit, do not impinge on that initial sense of being a teacher. At the same time, the messages that teacher educators may be attempting to convey, explicitly and implicitly, about what they consider is good teaching, or that they consider student teachers should know, will in some measure be oppressive, to the extent that they deny agency to student teachers. In an analogous way to a postmodern pedagogy for schools, a postmodern pedagogy in teacher education would: encourage the expression of difference; teach methods of critique of orthodoxies concerning mathematics and mathematics education; and encourage theorising about teaching and learning mathematics. It would find ways of confronting student teachers with their naive conceptions of teaching and with different theories about teaching which they might well have been espousing, through activities which bring these together (Crawford & Deer, 1993; Lerman, 1997). It would engage with the personal transitions in becoming a teacher, in terms of developing an identity as a person in that profession and with purposes appropriate to that role, by enabling them to constitute through articulation different ways of being, as a teacher. "Teaching is more difficult than learning; for only he who can truly learn - and as long as he can do it - can truly teach" (Heidegger, in Krell, 1993, p. 254).

Notes

(1)　My thanks to Peter Winbourne, Adam Vile and Anna Tsatsaroni for comments on an earlier draft.

(2)　This quote comes from the transcription by Anne Watson of a tape recording of a seminar with Rom Harré, held in Oxford University on December 6th 1997.

(3)　Along with Newman & Holzman (1993) I take the zpd to be the explanatory framework for learning as a whole, both in intentional settings, such as schooling, and in informal settings; in other words all socio-cultural milieus. In relation to classroom studies, the zpd is the classroom's.

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